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Mean-field dynamo due to fluctuating turbulent diffusivity

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In systems where the standard α effect is inoperative, one often explains the existence of mean magnetic fields by invoking the 'incoherent α effect', which appeals to fluctuations of the mean kinetic helicity. Previous studies, while considering fluctuations in the mean kinetic helicity, treated the mean turbulent kinetic energy as a constant, despite the fact that both these quantities involve second-order velocity correlations. The mean turbulent kinetic energy causes both turbulent diffusion and diamagnetic pumping of the mean magnetic field. In this work, we use a double-averaging procedure to analytically show that fluctuations of the mean turbulent kinetic energy (giving rise to η-fluctuations, where η is the turbulent diffusivity) can lead to the growth of a large-scale magnetic field even when the kinetic helicity is zero pointwise. Constraints on the operation of such a dynamo are expressed in terms of dynamo numbers that depend on the correlation length, correlation time, and strength of these fluctuations. In the white-noise limit, we find that these fluctuations reduce the overall turbulent diffusion, while also contributing a drift term which does not affect the growth of the field. We also study the effects of nonzero correlation time and anisotropy. Diamagnetic pumping, which arises due to inhomogeneities in the turbulent kinetic energy, leads to growing mean field solutions even when the η-fluctuations are isotropic. Our results suggest that fluctuations of the turbulent kinetic energy may be relevant in astrophysical contexts.

1. Introduction

Astrophysical magnetic fields are observed on galactic, stellar, and planetary scales (Brandenburg & Subramanian 2005; Jones 2011). Some stars even exhibit periodic magnetic cycles. The Earth itself has a dipolar magnetic field that shields it from the solar wind. Dynamo theory studies the mechanisms behind the generation and maintenance of these large-scale magnetic fields by fluid flows correlated at much smaller scales (Ruzmaikin *et al.* 1988; Brandenburg & Subramanian 2005; Jones 2011; Rincon 2019). Mean-field magnetohydrodynamics takes advantage of scale-separation to make the problem analytically tractable (Moffatt 1978; Krause & Rädler 1980).

The turbulent electromotive force, which is determined by correlations between the fluctuating velocity and magnetic fields, plays a crucial role in mean-field dynamo theory. For homogeneous and isotropic turbulence, using the quasilinear approximation, one can express the turbulent electromotive force in terms of the turbulent transport coefficients α

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(which is proportional to the mean kinetic helicity) and η (the turbulent diffusivity, which is proportional to the mean kinetic energy) when the magnetic field is weak (Moffatt 1978, chapter 7). The contribution of α, if nonzero, may cause growth of the mean magnetic field, while η always dissipates it when the turbulence is homogeneous.

Even when the mean kinetic helicity is zero, Kraichnan (1976) found that fluctuations of the kinetic helicity can suppress the turbulent diffusivity. If the fluctuations are strong or long-lived enough, the effective diffusivity may become negative, leading to growth of the large-scale magnetic field (Kraichnan 1976; Moffatt 1978, sec. 7.11; Singh 2016). This effect, usually referred to as the 'incoherent α effect', has also been studied in combination with shear (Sokolov 1997; Vishniac & Brandenburg 1997; Silant'ev 2000; Sridhar & Singh 2014). The 'incoherent α-shear dynamo' has been invoked (Brandenburg *et al.* 2008) to explain the generation of a large-scale magnetic field in simulations of nonhelical turbulence with background shear (Yousef *et al.* 2008; Singh & Jingade 2015).

To derive his result, Kraichnan (1976) used a process of double-averaging, where one first obtains the mean-field equations at some mesoscale, and then fluctuations of the mesoscale transport coefficients may lead to effects at some larger scale upon subsequent averaging. There are two viewpoints (not mutually exclusive) on the applicability of this method. One is that we require the system to have scale separation, such that the turbulent spectra peak at some small scale, while averaged quantities themselves fluctuate at some mesoscale, and then there exists an even larger scale where a magnetic field can grow (e.g. Moffatt 1978, p. 178). The other is to think of multiscale averaging as a renormalization procedure which tells us something about the contributions of higher moments of the velocity field to the turbulent transport coefficients (e.g. Moffatt 1983, sec. 11; Silant'ev 2000, p. 341). In support of the latter, we note that Knobloch (1977)† and Nicklaus & Stix (1988) have used a cumulant expansion to calculate the lowest-order corrections to the quasilinear approximation. In agreement with the results obtained by multiscale averaging, they find that the turbulent diffusivity is suppressed.

Regardless of one's viewpoint, it seems natural to wonder why fluctuations of the helicity should have a more privileged position than fluctuations of the kinetic energy. In simulations, it is found that fluctuations of α coexist with fluctuations of η (e.g. Brandenburg *et al.* 2008, fig. 10). While Silant'ev (1999, 2000) has considered fluctuations of the turbulent diffusivity, he has not included the effect of diamagnetic pumping (expulsion of the magnetic field from turbulent regions); the latter is a natural consequence of spatial variations of the turbulent kinetic energy, and thus cannot be ignored.

Here, we explore the effects of mesoscale fluctuations of the turbulent magnetic diffusivity, with nonzero correlation time, on the evolution of the large-scale magnetic field. The procedure we follow is the same as that of Singh (2016).

In section 2, we derive the evolution equation for the large-scale magnetic field, along with an expression for its growth rate, by using the quasilinear approximation. In section 3, we simplify the expression for the growth rate, assuming the fluctuations of η are isotropic. In section 4, we explain how the growth rate is modified by anisotropy. In section 5, we relate the growth in some regimes to a negative effective turbulent diffusivity. In section 6, we show how to estimate the dynamo numbers in astrophysical systems, taking the solar photosphere as an example. Finally, we discuss the implications of our results and possible future directions in section 7.

† Nicklaus & Stix (1988) point out some errors in this paper.

2. Derivation of the evolution equation and the growth rate

2.1. *Setup and assumptions*

The mean magnetic field, $\boldsymbol{B}$, evolves according to (e.g. Moffatt 1978, eq. 7.7)

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{V}\times\boldsymbol{B}+\boldsymbol{\mathcal{E}}) + \eta_m \nabla^2 \boldsymbol{B} \tag{2.1}$$

where $\boldsymbol{V}$ is the mean velocity; η_m is the microscopic magnetic diffusivity; and $\boldsymbol{\mathcal{E}}$, the turbulent electromotive force (EMF), is related to the correlation between the fluctuating velocity and magnetic fields. For weakly inhomogeneous nonhelical turbulence, the EMF is given by (Roberts & Soward 1975, eq. 3.11)

$$\boldsymbol{\mathcal{E}} = -\frac{1}{2}\nabla\eta\times\boldsymbol{B} - \eta\nabla\times\boldsymbol{B} \tag{2.2}$$

where η is the turbulent diffusivity (related to the turbulent kinetic energy in simple closures). We note that Silant'ev (1999, 2000) did not consider the first term above. For stratified turbulence, or in the presence of small-scale magnetic fields, additional terms arise (Vainshtein & Kichatinov 1983), but we ignore those effects in this work. As mentioned in the introduction, the effect of helical turbulence (α) has been extensively studied, so we restrict ourselves to turbulence that is nonhelical pointwise.

Although the mean-field approach does not formally require scale-separation, we associate averages with length/time scales for clarity of exposition. Let us assume that η fluctuates at length/time scales (henceforth referred to as the mesoscales) much larger than than the scales at which the turbulent velocity fluctuates. We employ a double-averaging approach (Kraichnan 1976; Singh 2016), in which we treat η (at the mesoscale) as a stochastic scalar field which is a function of both position and time (i.e. $\eta = \eta(\boldsymbol{x}, t)$). For any mesoscale quantity $\square$, we use $\langle\square\rangle$ and $\overline{\square}$ to denote its averages at the larger scale. We assume this average satisfies Reynolds' rules (e.g. Monin & Yaglom 1971, sec. 3.1).

If we set the mean velocity to zero, ignore the microscopic diffusivity (which is usually much smaller than the turbulent diffusivity in the systems of interest), and use equation 2.2, we can write equation 2.1 as

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times\left(-\frac{1}{2}\nabla\eta\times\boldsymbol{B} - \eta\nabla\times\boldsymbol{B}\right) \tag{2.3}$$

One of the most widely used closures in dynamo theory is the quasilinear approximation (also called the first-order smoothing approximation, FOSA; or the second-order correlation approximation, SOCA) (e.g. Moffatt 1978, sec. 7.5; Krause & Rädler 1980, sec. 4.3) The quasilinear approximation is rigorously valid only when either the magnetic Reynolds number (the ratio of the diffusive to the advective timescale) or the Strouhal number (the ratio of the velocity correlation time to its turnover time) are small (Krause & Rädler 1980, p. 49). The former is never small in the astrophysical systems of interest, while it is unclear if the latter is small. Nevertheless, in the context of mean-field dynamo theory, the quasilinear approximation often remains qualitatively correct well outside its domain of formal validity. More complicated closures such as the EDQNM closure (e.g. Pouquet *et al.* 1976) and the DIA (Kraichnan 1977) are extremely difficult to work with.

In section 2.3, we assume the fluctuations of η are statistically homogeneous, stationary, and separable in order to obtain an integro-differential equation for the large-scale magnetic field. In section 2.4, we simplify this equation by assuming the fluctuations of η are white noise, while in section 2.5, we also keep terms linear in the correlation time of η.

2.2. *Evolution equation in Fourier space*

We now move to Fourier space with

$$\widetilde{f}(\boldsymbol{k},t) \equiv \int \frac{\mathrm{d}\boldsymbol{x}}{(2\pi)^3}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} f(\boldsymbol{x},t) \tag{2.4}$$

in which case the convolution theorem takes the form

$$\int \frac{\mathrm{d}\boldsymbol{x}}{(2\pi)^3}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} f(\boldsymbol{x})g(\boldsymbol{x}) = \int \mathrm{d}\boldsymbol{p}\, \widetilde{f}(\boldsymbol{p})\widetilde{g}(\boldsymbol{k}-\boldsymbol{p}) \tag{2.5}$$

Equation 2.3 then becomes (omitting the temporal arguments whenever there is no ambiguity)

$$\frac{\partial \widetilde{\boldsymbol{B}}(\boldsymbol{k})}{\partial t} = \int \mathrm{d}\boldsymbol{p}\; \boldsymbol{k} \times \left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2}\right) \times \left(\widetilde{\eta}(\boldsymbol{k}-\boldsymbol{p})\, \widetilde{\boldsymbol{B}}(\boldsymbol{p})\right)\right] \tag{2.6}$$

Taking the average of the above, we obtain

$$\frac{\partial}{\partial t}\left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k})\right\rangle = \int \mathrm{d}\boldsymbol{p}\; \boldsymbol{k} \times \left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2}\right) \times \left\{\langle\widetilde{\eta}(\boldsymbol{k}-\boldsymbol{p})\rangle \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{p})\right\rangle + \left\langle \widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p})\, \widetilde{\boldsymbol{b}}(\boldsymbol{p})\right\rangle\right\}\right] \tag{2.7}$$

where we have split the mesoscale fields into their mean and fluctuating parts, i.e. $\widetilde{\boldsymbol{B}} = \left\langle \widetilde{\boldsymbol{B}}\right\rangle + \widetilde{\boldsymbol{b}}$ and $\widetilde{\eta} = \langle\widetilde{\eta}\rangle + \widetilde{\mu}$. We write the equation for $\widetilde{\boldsymbol{b}}$ as

$$\begin{aligned}\frac{\partial \widetilde{\boldsymbol{b}}(\boldsymbol{k})}{\partial t} = \int \mathrm{d}\boldsymbol{p}\; \boldsymbol{k} \times \Bigg[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2}\right) \times \Bigg(& \langle\widetilde{\eta}(\boldsymbol{k}-\boldsymbol{p})\rangle\, \widetilde{\boldsymbol{b}}(\boldsymbol{p}) + \widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p}) \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{p})\right\rangle \\ & + \widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p})\, \widetilde{\boldsymbol{b}}(\boldsymbol{p}) - \left\langle \widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p})\, \widetilde{\boldsymbol{b}}(\boldsymbol{p})\right\rangle \Bigg)\Bigg]\end{aligned} \tag{2.8}$$

We now apply the quasilinear approximation, where the equations for the fluctuating fields are truncated by keeping only terms which are at most linear in the fluctuating fields. We then obtain

$$\frac{\partial \widetilde{\boldsymbol{b}}(\boldsymbol{k})}{\partial t} = \int \mathrm{d}\boldsymbol{p}\; \boldsymbol{k} \times \left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2}\right) \times \left(\langle\widetilde{\eta}(\boldsymbol{k}-\boldsymbol{p})\rangle\, \widetilde{\boldsymbol{b}}(\boldsymbol{p}) + \widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p}) \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{p})\right\rangle\right)\right] \tag{2.9}$$

2.3. *Homogeneity and separability*

To simplify the preceding expression, we assume that the moments of $\eta(\boldsymbol{x},t)$ are statistically homogeneous and stationary. Further, we assume that $\langle\mu(\boldsymbol{x},\tau_1)\mu(\boldsymbol{y},\tau_2)\rangle$ can be written as the product of a temporal correlation function and a spatial correlation function. In Fourier space, these assumptions can be expressed as

$$\langle\widetilde{\eta}(\boldsymbol{k},t)\rangle = \overline{\eta}\,\delta(\boldsymbol{k}) \tag{2.10a}$$

$$\langle\widetilde{\mu}(\boldsymbol{p},\tau_1)\widetilde{\mu}(\boldsymbol{q},\tau_2)\rangle = \widetilde{Q}(\boldsymbol{p})\, S(\tau_1-\tau_2)\, \delta(\boldsymbol{p}+\boldsymbol{q}) \tag{2.10b}$$

For S, we require

$$2\int_0^\infty S(t)\,\mathrm{d}t = 1 \tag{2.11}$$

and define the correlation time of η as

$$\tau_\eta \equiv 2\int_0^\infty tS(t)\,\mathrm{d}t \tag{2.12}$$

We can then write

$$\int \mathrm{d}\boldsymbol{p}\ \boldsymbol{k} \times \left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2} \right) \times \left(\langle \widetilde{\eta}(\boldsymbol{k}-\boldsymbol{p}) \rangle\, \widetilde{\boldsymbol{b}}(\boldsymbol{p}) \right) \right] = -\overline{\eta} k^2 \widetilde{\boldsymbol{b}}(\boldsymbol{k}) \tag{2.13}$$

and

$$\int \mathrm{d}\boldsymbol{p}\ \boldsymbol{k} \times \left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2} \right) \times \left(\langle \widetilde{\eta}(\boldsymbol{k}-\boldsymbol{p}) \rangle \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{p}) \right\rangle \right) \right] = -\overline{\eta} k^2 \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k}) \right\rangle \tag{2.14}$$

Using equation 2.13, equation 2.9 can be written as

$$\frac{\partial \widetilde{\boldsymbol{b}}(\boldsymbol{k})}{\partial t} = -\overline{\eta} k^2 \widetilde{\boldsymbol{b}}(\boldsymbol{k}) + \int \mathrm{d}\boldsymbol{p}\ \boldsymbol{k} \times \left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2} \right) \times \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{p}) \right\rangle \right] \widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p}) \tag{2.15}$$

which gives us

$$\begin{aligned} \widetilde{\boldsymbol{b}}(\boldsymbol{k},t) = & \int_0^t \mathrm{d}\tau \int \mathrm{d}\boldsymbol{p}\ e^{-\overline{\eta} k^2 (t-\tau)}\ \boldsymbol{k} \times \left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2} \right) \times \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{p},\tau) \right\rangle \right] \widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p},\tau) \\ & + \widetilde{\boldsymbol{b}}(\boldsymbol{k},0) \end{aligned} \tag{2.16}$$

We assume that the initial fluctuations of the mesoscale magnetic field are uncorrelated with μ. Using the above along with equation 2.10*b*, we can write

$$\left\langle \widetilde{\mu}(\boldsymbol{q},t) \widetilde{\boldsymbol{b}}(\boldsymbol{k},t) \right\rangle = \int_0^t \mathrm{d}\tau\, e^{-\overline{\eta} k^2 (t-\tau)}\ \boldsymbol{k} \times \left[\left(\boldsymbol{k} + \frac{\boldsymbol{q}}{2} \right) \times \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k}+\boldsymbol{q},\tau) \right\rangle \right] \widetilde{Q}(\boldsymbol{q})\, S(t-\tau) \tag{2.17}$$

Putting the above in equation 2.7 gives us an equation for $\left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k}+\boldsymbol{q},\tau) \right\rangle$. However, this is an integro-differential equation which is difficult to solve in general. The resulting equation can be simplified by assuming τ_η is small. In section 2.4, we assume $\tau_\eta = 0$ and simplify the evolution equation for the large-scale magnetic field. In section 2.5, we simplify the evolution equation neglecting $O(\tau_\eta^2)$ terms.

2.4. *Evolution equation with white-noise fluctuations*

Assuming $S(t) = \delta(t)$, we write equation 2.17 as

$$\left\langle \widetilde{\mu}(\boldsymbol{q},t) \widetilde{\boldsymbol{b}}(\boldsymbol{k},t) \right\rangle = \frac{1}{2}\, \boldsymbol{k} \times \left[\left(\boldsymbol{k} + \frac{\boldsymbol{q}}{2} \right) \times \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k}+\boldsymbol{q},t) \right\rangle \right] \widetilde{Q}(\boldsymbol{q}) \tag{2.18}$$

Recalling that $\boldsymbol{k} \cdot \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k},t) \right\rangle = 0$, we can use the above to write

$$\begin{aligned} & \int \mathrm{d}\boldsymbol{p}\ \boldsymbol{k} \times \left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2} \right) \times \left\langle \widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p},t) \widetilde{\boldsymbol{b}}(\boldsymbol{p},t) \right\rangle \right] \\ & = \int \mathrm{d}\boldsymbol{s}\ \frac{1}{8} \widetilde{Q}(\boldsymbol{s}) \left\{ 4k^4 - 8k^2 \boldsymbol{k}\cdot\boldsymbol{s} + 3\,(\boldsymbol{k}\cdot\boldsymbol{s})^2 + 2k^2 s^2 - s^2 \boldsymbol{k}\cdot\boldsymbol{s} \right\} \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k},t) \right\rangle \end{aligned} \tag{2.19}$$

where $\boldsymbol{s} \equiv \boldsymbol{k} - \boldsymbol{p}$. Defining

$$A^{(0)} \equiv Q(\boldsymbol{0})\,,\ A_i^{(1)} \equiv \left. \frac{\partial Q(\boldsymbol{y})}{\partial y_i} \right|_{\boldsymbol{y}=\boldsymbol{0}},\ A_{ij}^{(2)} \equiv \left. \frac{\partial^2 Q(\boldsymbol{y})}{\partial y_i \partial y_j} \right|_{\boldsymbol{y}=\boldsymbol{0}},\ A_i^{(3)} \equiv \left. \frac{\partial^3 Q(\boldsymbol{y})}{\partial y_i \partial y_j \partial y_j} \right|_{\boldsymbol{y}=\boldsymbol{0}} \tag{2.20}$$

we write

$$
\begin{aligned}
&\int \mathrm{d}\boldsymbol{p}\, \boldsymbol{k} \times \left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2} \right) \times \left\langle \widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p},t) \widetilde{\boldsymbol{b}}(\boldsymbol{p},t) \right\rangle \right] \\
&= \frac{1}{8} \left\{ 4A^{(0)} k^4 - 8iA_i^{(1)} k^2 k_i - 3A_{ij}^{(2)} k_i k_j - 2A_{ii}^{(2)} k^2 + iA_i^{(3)} k_i \right\} \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k},t) \right\rangle
\end{aligned}
\tag{2.21}
$$

Putting this in equation 2.7 and using equation 2.14, we obtain

$$
\frac{\partial}{\partial t} \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k}) \right\rangle = \left(-\overline{\eta} k^2 + g(\boldsymbol{k}) \right) \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k}) \right\rangle + i h(\boldsymbol{k}) \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k}) \right\rangle \tag{2.22}
$$

where

$$
g(\boldsymbol{k}) \equiv -\frac{3}{8} A_{ij}^{(2)} k_i k_j - \frac{1}{4} A_{ii}^{(2)} k^2 + \frac{1}{2} A^{(0)} k^4 \tag{2.23a}
$$

$$
h(\boldsymbol{k}) \equiv \frac{1}{8} A_i^{(3)} k_i - A_i^{(1)} k^2 k_i \tag{2.23b}
$$

We see that $g(\boldsymbol{k})$ describes corrections (including hyperdiffusion) to the turbulent diffusivity, while the term involving $h(\boldsymbol{k})$ describes advection of the large-scale magnetic field with an effective velocity $A_i^{(3)}/8 - A_i^{(1)} k^2$.

To aid the interpretation of equation 2.22, we note that if the spatial correlation function of the fluctuations of η is an isotropic Gaussian (see appendix A), we can write

$$
\frac{\partial}{\partial t} \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k}) \right\rangle = -\left(\overline{\eta} - \frac{9}{8} \beta \right) k^2 \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k}) \right\rangle + O(k^4) \,, \tag{2.24}
$$

where $\beta \equiv A^{(0)}/l_c^2 > 0$ represents the diffusivity arising from fluctuations of η with a correlation length l_c. Thus, we find that fluctuations of η reduce the turbulent diffusion of the large-scale magnetic field.

2.5. *Evolution equation with nonzero correlation time*

We expand

$$
\left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{p},\tau) \right\rangle = \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{p},t) \right\rangle - (t-\tau) \frac{\partial}{\partial t} \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{p},t) \right\rangle + O((t-\tau)^2) \tag{2.25}
$$

The idea is that when we substitute this into equation 2.17, assume $t \gg \tau_\eta$, and perform the time integral, the powers of $(t-\tau)$ become powers of τ_η. The convergence of this series requires that the large-scale magnetic field vary on a timescale much larger than τ_η. Note that on the RHS of the above, we can neglect $O(t-\tau)$ contributions to $\frac{\partial}{\partial t}\left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{p},t) \right\rangle$ and use equation 2.22. Similarly, we can expand

$$
\exp\left(-\overline{\eta} k^2 (t-\tau) \right) = 1 - (t-\tau)\, \overline{\eta} k^2 + O((t-\tau)^2) \tag{2.26}
$$

We then write equation 2.17 as

$$
\left\langle \widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p},t) \widetilde{\boldsymbol{b}}(\boldsymbol{p},t) \right\rangle = \widetilde{Q}(\boldsymbol{k}-\boldsymbol{p})\ \boldsymbol{p} \times \left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2} \right) \times \mathscr{B}(\boldsymbol{k},t) \right] \tag{2.27}
$$

where

$$
\mathscr{B}(\boldsymbol{k},t) \equiv \frac{1}{2} \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k},t) \right\rangle - \frac{\tau_\eta}{2} \frac{\partial}{\partial t} \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k},t) \right\rangle - \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k},t) \right\rangle \frac{\tau_\eta}{2} \overline{\eta} k^2 \tag{2.28}
$$

Using equation 2.22, we write

$$\mathcal{B}(\boldsymbol{k},t) = \frac{1}{2}\left\langle\widetilde{\boldsymbol{B}}(\boldsymbol{k},t)\right\rangle - \frac{\tau_\eta}{2}\, g(\boldsymbol{k})\left\langle\widetilde{\boldsymbol{B}}(\boldsymbol{k},t)\right\rangle - \frac{i\tau_\eta}{2}\, h(\boldsymbol{k})\left\langle\widetilde{\boldsymbol{B}}(\boldsymbol{k},t)\right\rangle \tag{2.29}$$

We note that $\boldsymbol{k}\cdot\mathcal{B}(\boldsymbol{k},t) = 0$ and use equations 2.27 and 2.29 to write

$$\begin{aligned}\int \mathrm{d}\boldsymbol{p}\;\boldsymbol{k}\times\left[\left(\frac{\boldsymbol{k}+\boldsymbol{p}}{2}\right)\times\left\langle\widetilde{\mu}(\boldsymbol{k}-\boldsymbol{p},t)\widetilde{\boldsymbol{b}}(\boldsymbol{p},t)\right\rangle\right]&\\ = \left[g(\boldsymbol{k}) + ih(\boldsymbol{k})\right]\left[1-\tau_\eta g(\boldsymbol{k}) - i\tau_\eta h(\boldsymbol{k})\right]\left\langle\widetilde{\boldsymbol{B}}(\boldsymbol{k},t)\right\rangle&\end{aligned} \tag{2.30}$$

where h and g are defined in equations 2.23. Putting this in equation 2.7 and using equation 2.14, we write

$$\frac{\partial}{\partial t}\left\langle\widetilde{\boldsymbol{B}}(\boldsymbol{k})\right\rangle = \left[g(\boldsymbol{k}) + ih(\boldsymbol{k})\right]\left[1-\tau_\eta g(\boldsymbol{k}) - i\tau_\eta h(\boldsymbol{k})\right]\left\langle\widetilde{\boldsymbol{B}}(\boldsymbol{k})\right\rangle - \overline{\eta}k^2\left\langle\widetilde{\boldsymbol{B}}(\boldsymbol{k})\right\rangle + O(\tau_\eta^2) \tag{2.31}$$

2.6. *Growth rate of the large-scale magnetic field*

Let us now focus on the problem of whether a particular Fourier mode of the large-scale magnetic field grows or decays. We assume $\left\langle\widetilde{\boldsymbol{B}}(\boldsymbol{k},t)\right\rangle \propto \exp(\lambda t)$. Plugging this into equation 2.31 and taking its real part, we find

$$\mathrm{Re}(\lambda) = -\overline{\eta}k^2 + g(\boldsymbol{k}) - \tau_\eta\left[g(\boldsymbol{k})\right]^2 + \tau_\eta\left[h(\boldsymbol{k})\right]^2 + O(\tau_\eta^2) \tag{2.32}$$

where h and g are defined in equations 2.23. From the fact that above, only $[g(\boldsymbol{k})]^2$ contains a k^8 term, we can see that the growth rate always becomes negative for large-enough k (small-enough scales) as long as $\tau_\eta \neq 0$. Note that while in the white-noise case, $h(\boldsymbol{k})$ only contributed a drift term, it now affects the growth rate as well.

Since we assumed the large-scale magnetic field varies on timescales much larger than τ_η, our derivation is self-consistent only when $\left|\tau_\eta\lambda\right| \ll 1$.

3. Dynamo numbers when the fluctuations are isotropic

If $Q(\boldsymbol{y})$ is isotropic,† we can write the quantities defined in equation 2.20 as

$$A_i^{(1)} = 0\,,\quad A_{ij}^{(2)} = \delta_{ij}\frac{A_{kk}^{(2)}}{3}\,,\quad A_i^{(3)} = 0 \tag{3.1}$$

so that

$$h(\boldsymbol{k}) = 0\,,\quad g(\boldsymbol{k}) = \frac{4A^{(0)}k^4 - 3k^2A_{mm}^{(2)}}{8} \tag{3.2}$$

Equation 2.32 can then be written as

$$\begin{aligned}\mathrm{Re}(\lambda) = &-k^2\left(\overline{\eta} + \frac{3A_{mm}^{(2)}}{8}\right) + k^4\left(\frac{A^{(0)}}{2} - \frac{9\tau_\eta}{64}\left[A_{mm}^{(2)}\right]^2\right)\\ &+\frac{3\tau_\eta A^{(0)}A_{mm}^{(2)}k^6}{8} - \frac{\tau_\eta\left[A^{(0)}\right]^2k^8}{4}\end{aligned} \tag{3.3}$$

† Mirror symmetry is not required.

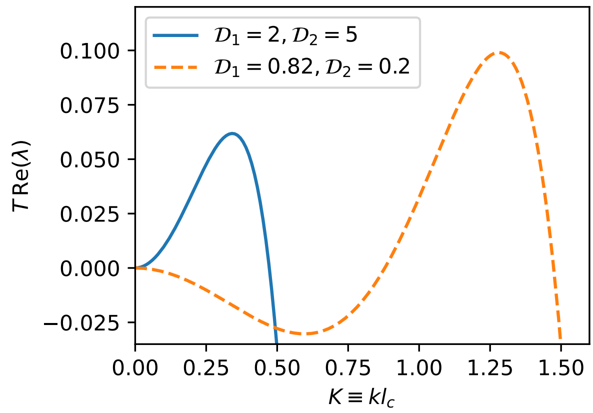

Figure 1: The mode growth rate ($T\,\mathrm{Re}(\lambda)$, equation 3.5) as a function of the wavenumber for two combinations of $\mathcal{D}_1$ and $\mathcal{D}_2$.

If we further define‡

$$\mathcal{D}_1 \equiv -\frac{3A^{(2)}_{mm}}{8\overline{\eta}}\,,\; \mathcal{D}_2 \equiv \frac{9\tau_\eta}{32}\frac{\left[A^{(2)}_{mm}\right]^2}{A^{(0)}}\,,\; l_c \equiv \sqrt{-\frac{3A^{(0)}}{A^{(2)}_{ii}}}\,,\; K \equiv kl_c\,,\; T \equiv \frac{l_c^2}{\overline{\eta}} \tag{3.4}$$

we can write equation 3.3 as

$$T\,\mathrm{Re}(\lambda) = -K^2\left(1-\mathcal{D}_1\right) + \frac{4\mathcal{D}_1 K^4}{9}\left(1-\mathcal{D}_2\right) - \frac{32\mathcal{D}_1\mathcal{D}_2 K^6}{81} - \frac{64\mathcal{D}_1\mathcal{D}_2 K^8}{729} \tag{3.5}$$

In appendix A, we express the dynamo numbers in terms of more observationally relevant quantities by assuming a particular form for the correlation function Q.

Figure 1 shows the growth rate (equation 3.5) for two sets of dynamo numbers. We see that depending on the parameters, the growth rate may peak at large scales or at small scales.

To understand the qualitative behaviour of equation 3.5, we can schematically write it as

$$\mathrm{Re}(\lambda) = \begin{cases} -k^2 - k^4 - k^6 - k^8 & ;\ \mathcal{D}_2 > 1,\ \mathcal{D}_1 < 1 \\ -k^2 + k^4 - k^6 - k^8 & ;\ \mathcal{D}_2 < 1,\ \mathcal{D}_1 < 1 \\ k^2 + k^4 - k^6 - k^8 & ;\ \mathcal{D}_2 < 1,\ \mathcal{D}_1 > 1 \\ k^2 - k^4 - k^6 - k^8 & ;\ \mathcal{D}_2 > 1,\ \mathcal{D}_1 > 1 \end{cases} \tag{3.6}$$

In the first case, $\mathrm{Re}(\lambda)$ is always negative, and so there is no dynamo. In the last two cases, $\mathrm{Re}(\lambda)$ is positive for small k and becomes negative for large wavenumbers. In the second regime, it seems to be difficult to say anything concrete (depending on the values of the coefficients, one can either have growth in a range of wavenumbers or growth nowhere).

Since 3.5 is a polynomial in K, one can easily solve for its extrema. In figure 2, we show the dynamo growth rate (where positive) and the wavenumber of the resulting large-scale field, as a function of $\mathcal{D}_1$ and $\mathcal{D}_2$.

If we drop the $O(K^6)$ terms in equation 3.5 (this does not change the qualitative behaviour when $\mathcal{D}_2 > 1$), we can estimate that that if $\mathcal{D}_1 > 1$, the growth rate attains a maximum value

‡ If the correlation function attains a maximum at zero separation, $A^{(2)}_{mm} < 0$. This implies $\mathcal{D}_1 > 0$.

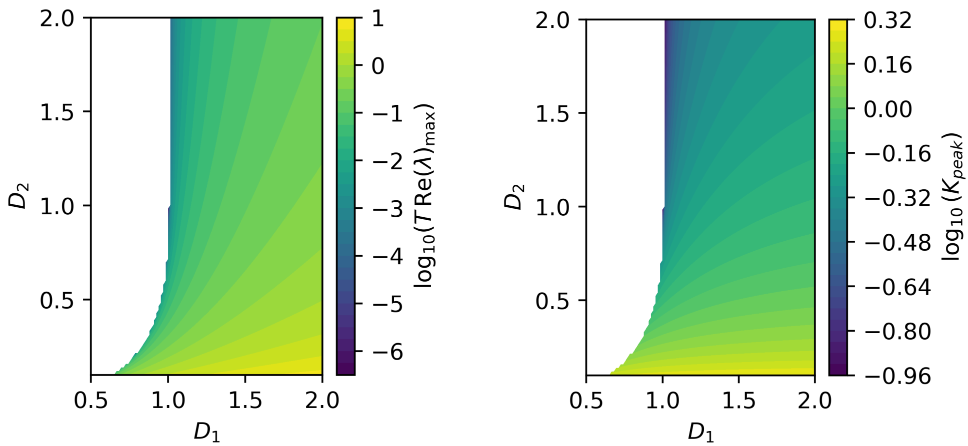

Figure 2: Left: The peak growth rate ($T\,\mathrm{Re}(\lambda)$, equation 3.5). In the white regions, the growth rate is negative for all K. Right: The wavenumber (K) at which the growth rate peaks. Recall that for a mode with wavelength l_c, the wavenumber would be $K = 2\pi \approx 10^{0.8}$.

at K_{peak}, where

$$K_{\mathrm{peak}} \approx \sqrt{\frac{9\,(\mathcal{D}_1 - 1)}{8\mathcal{D}_1\,(\mathcal{D}_2 - 1)}}\,, \qquad [T\,\mathrm{Re}(\lambda)]_{\mathrm{max}} \approx \frac{9\,(\mathcal{D}_1 - 1)^2}{16\mathcal{D}_1\,(\mathcal{D}_2 - 1)} \tag{3.7}$$

Broadly speaking, there are two kinds of regimes in which the dynamo is excited. One, $\mathcal{D}_1 > 1$, corresponds to the fluctuations being strong enough that the effective diffusivity itself becomes negative (but the growth itself is still cut off at small scales due to higher-order terms). The other, $\mathcal{D}_2 < 1$ (with $\mathcal{D}_1$ also < 1), corresponds to growth with the effective diffusion remaining positive; one can see, however, from figure 2 that this growth happens at smaller scales than in the other regime (but may still be at scales larger than l_c). While $\mathcal{D}_2 \ll 1$ can formally lead to growing solutions regardless of the value of $\mathcal{D}_1$, the growth then occurs at scales $\lesssim l_c$.

4. The effect of anisotropy

Although we have not done so so far, it seems natural to assume that the temporal correlation function S, that appears in equation 2.10*b*, is even. This would allow one to take $\int_{-\infty}^{\infty} S(t)\,\mathrm{d}t = 1$ and define the correlation time of η as $\tau_\eta \equiv \int_{-\infty}^{\infty} |t|\,S(t)\,\mathrm{d}t$.

Because μ is a scalar, assuming its double correlation is invariant under time-reversal immediately implies $\widetilde{Q}(\boldsymbol{k}) = \widetilde{Q}(-\boldsymbol{k})$. We then conclude that

$$A_i^{(1)} = A_i^{(3)} = h(\boldsymbol{k}) = 0 \tag{4.1}$$

when the fluctuations of η are separable, homogeneous, stationary, and time-reversal-invariant; this holds even without assuming that the fluctuations of η are isotropic! We now study the dynamo assuming the double correlation of μ is time-reversal invariant and anisotropic.

Let us choose the coordinate axes $1, 2, 3$ to be along the principal axes of the matrix $A^{(2)}$ (defined in equation 2.20), with the corresponding eigenvalues being $-a_1$, $-a_2$, and $-a_3$ (such

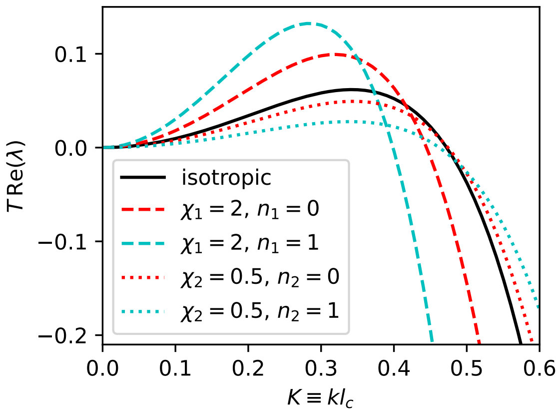

Figure 3: The mode growth rate ($T\,\mathrm{Re}(\lambda)$, equation 4.6) as a function of the wavenumber. In all the cases, we have taken $\mathcal{D}_1 = 2$ and $\mathcal{D}_2 = 5$. Other parameters not mentioned in the legend have been set to zero.

that $a_1 \geqslant a_3 \geqslant a_2$). By analogy with equation 3.4, one can define the correlation length along each axis as $l_c^{(i)} \equiv \sqrt{A^{(0)}/a_i}$. It is physically reasonable to assume $Q(\boldsymbol{y})$ attains a local maximum at the origin, and that its correlation length is finite. This means $a_1, a_2, a_3 > 0$.

Analogous to equation 3.4, we define

$$\mathcal{D}_1 \equiv \frac{9a_3}{8\overline{\eta}}\,,\ \mathcal{D}_2 \equiv \frac{81\tau_\eta a_3^2}{32A^{(0)}}\,,\ l_c \equiv \sqrt{\frac{A^{(0)}}{a_3}}\,,\ \boldsymbol{K} \equiv \boldsymbol{k}l_c\,,\ T \equiv \frac{l_c^2}{\overline{\eta}} \tag{4.2}$$

We also define the new quantities

$$\chi_1 \equiv \frac{a_1}{a_3} - 1\,,\ \chi_2 \equiv 1 - \frac{a_2}{a_3}\,,\ n_1 \equiv \frac{|K_1|}{K}\,,\ n_2 \equiv \frac{|K_2|}{K} \tag{4.3}$$

and a modified dynamo number

$$\widetilde{\mathcal{D}}_1 \equiv \mathcal{D}_1\left[1 + \frac{\chi_1}{9}\left(2 + 3n_1^2\right) - \frac{\chi_2}{9}\left(2 + 3n_2^2\right)\right] \tag{4.4}$$

Since $0 \leqslant \chi_2 < 1$, $0 \leqslant \chi_1 < \infty$, and $0 \leqslant n_1, n_2 \leqslant 1$, we find that $\widetilde{\mathcal{D}}_1 > 0$. We write $g(\boldsymbol{k})$ (equation 2.23) as

$$Tg(\boldsymbol{k}) = \widetilde{\mathcal{D}}_1 K^2 + \frac{4\mathcal{D}_1}{9}K^4 \tag{4.5}$$

Noting that $\tau_\eta/T = 4\mathcal{D}_2/(9\mathcal{D}_1)$, one can substitute equation 4.5 in equation 2.32 to obtain the following expression for the growth rate:

$$T\,\mathrm{Re}(\lambda) = K^2\left(\widetilde{\mathcal{D}}_1 - 1\right) + \frac{4\mathcal{D}_1K^4}{9}\left(1 - \frac{\mathcal{D}_2\widetilde{\mathcal{D}}_1^2}{\mathcal{D}_1^2}\right) - \frac{32\mathcal{D}_2\widetilde{\mathcal{D}}_1K^6}{81} - \frac{64\mathcal{D}_1\mathcal{D}_2K^8}{729} \tag{4.6}$$

As expected, this reduces to equation 3.5 on setting $\widetilde{\mathcal{D}}_1 = \mathcal{D}_1$. Replacing $\mathcal{D}_1 \to \widetilde{\mathcal{D}}_1$ and $\mathcal{D}_2 \to \mathcal{D}_2\widetilde{\mathcal{D}}_1^2/\mathcal{D}_1^2$, our comments in section 3 on the qualitative behaviour of equation 3.5 also apply to this equation. Unlike in the isotropic case, the growth rate now depends on the direction of $\boldsymbol{K}$ through the direction cosines n_1 and n_2. Figure 3 shows the growth rate as a function of the wavenumber for various parameter values.

5. Suppression of turbulent diffusion

Neglecting terms with more than two spatial derivatives of $\langle \boldsymbol{B} \rangle$, equation 2.31 can be written as

$$\begin{aligned}\frac{\partial}{\partial t}\left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k})\right\rangle = &-\overline{\eta}k^2\left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k})\right\rangle - \frac{1}{8}\left(3k_m k_n A^{(2)}_{mn} + 2k^2 A^{(2)}_{mm}\right)\left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k})\right\rangle \\ &+ \frac{i}{8}\, k_m A^{(3)}_m \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k})\right\rangle + \frac{\tau_\eta}{64}\, k_m k_i A^{(3)}_m A^{(3)}_i \left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k})\right\rangle + O(k^3)\end{aligned} \tag{5.1}$$

Following the reasoning used in section 4 for $\widetilde{\mathcal{D}}_1$, one can see that the coefficient of $\left\langle \widetilde{\boldsymbol{B}}(\boldsymbol{k})\right\rangle$ in the second term above is always positive as long as the spatial correlation function of the η-fluctuations attains a maximum at zero separation; the turbulent diffusion is suppressed. This can be seen more clearly in equation 2.24, which assumes a particular form for the spatial correlation function. The third term is just an advection term, analogous to 'Moffat drift' (Moffatt 1978, sec. 7.11). The fourth term can never be negative, and is nonzero only when the fluctuations of η are anisotropic and not invariant under time-reversal. As noted in section 3, the higher powers of k neglected in equation 5.1 can cause growth of the large-scale magnetic field even when the effective diffusivity is positive. They also ensure that the growth rate becomes negative at small scales.

It may seem counter-intuitive that a dissipative term (η) at the mesoscale leads to a dynamo at larger scales, but it must be noted that in addition to dissipation, η also contributes an effective advection term (usually referred to as 'diamagnetic pumping'; see equation 2.2) when spatial variations at the mesoscale are properly accounted for.

6. Estimates of the dynamo numbers

Unfortunately, fluctuations of the turbulent diffusivity in astrophysical systems are not sufficiently constrained by observations. The situation in the solar photosphere is comparatively better, as observations of granulation give us an idea of the order of magnitude of various quantities. To make crude estimates, we use equation A 4 which assumes a specific form for the correlation function of η.

Let us assume $l_c = 3\,\mathrm{Mm}$ (peak of the granulation's power spectrum as observed by Roudier & Muller 1986, fig. 2) and $\tau_\eta = 400\,\mathrm{s}$ (granule lifetime measured by Bahng & Schwarzschild 1961). The turbulent diffusivity in the photosphere is a scale-dependent quantity, which is moreover not very well constrained (Abramenko *et al.* 2011, fig. 10). For the length scales of interest, it is not unreasonable to take $\overline{\eta} = 600\,\mathrm{km^2\,s^{-1}}$. Let us also assume $f = 0.1$ ($f \equiv \left\langle \mu^2 \right\rangle / \overline{\eta}^2$). We then find $\mathcal{D}_1 \approx 6 \times 10^{-3}$ and $\mathcal{D}_2 \approx 4 \times 10^{-4}$. These estimates appear to rule out the operation of such a dynamo in the solar photosphere. However, we note that assuming slightly different values of l_c and τ_η brings the dynamo numbers to within the regime where a large-scale field can be generated; for example, taking $l_c = 300\,\mathrm{km}$ and $\tau_\eta = 900\,\mathrm{s}$ gives us $\mathcal{D}_1 \approx 1.4$ and $\mathcal{D}_2 \approx 18$. The dynamo numbers are also affected by uncertainties in f. Further, anisotropy can have a significant effect on the growth rates. Better estimates of the dynamo numbers would require measurements of the spatiotemporal correlation and strength of fluctuations of the turbulent diffusivity (or the kinetic energy) in the solar photosphere.

7. Conclusions

We have used a double-averaging procedure and found that just like helicity fluctuations, fluctuations of the turbulent kinetic energy can drive the growth of a large-scale magnetic field. This dynamo is driven by diamagnetic pumping.

In the white-noise limit, we have found that η-fluctuations cause a reduction in the overall turbulent diffusion, while also contributing a drift term which does not affect the growth of the field. We have then explored effects of nonzero correlation times and found the possibility of growing mean field solutions with the overall turbulent diffusion remaining positive. When the fluctuations are isotropic, the growth rate of a particular Fourier mode of the large-scale magnetic field depends on the magnitude of its wavevector and on two dynamo numbers. Anisotropy leads to a dependence on, among other things, the direction of the wavevector.

We have studied the conditions under which this new dynamo can operate. However, the lack of precise estimates of the quantities involved makes it hard to conclusively rule out or support the resulting dynamo in various astrophysical scenarios. Given the prevalence of shear in astrophysical systems, an obvious extension of the current work would be to study the implications, for a large-scale magnetic field, of fluctuations of the turbulent kinetic energy in a shearing background. Since inhomogeneities in the density and in the small-scale magnetic energy also give rise to pumping (Vainshtein & Kichatinov 1983), we expect them to have effects similar to those described here.

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Author contributions. KG and NS conceptualized the research, interpreted the results, and wrote the paper. KG performed the calculations.

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Appendix A. Dynamo numbers for a simple correlation function

To physically interpret $\mathcal{D}_1$ and $\mathcal{D}_2$ (defined in equation 3.4), it is helpful to explicitly write them out for a specific functional form of Q (see equation 2.10*b*). We take

$$Q(\boldsymbol{y}) = C\exp\left(-\frac{|\boldsymbol{y}|^2}{2l_c^2}\right)\,,\qquad S(t) = \frac{1}{2\tau_\eta}\exp\left(-\frac{|t|}{\tau_\eta}\right) \tag{A 1}$$

which gives us

$$A^{(0)} = C > 0\,,\quad A_i^{(1)} = 0\,,\quad A_{ij}^{(2)} = -\frac{C}{l_c^2}\,\delta_{ij}\,,\quad A_i^{(3)} = 0 \tag{A 2}$$

If we define

$$\tilde{\tau} \equiv \frac{\tau_\eta}{T} = \frac{\tau_\eta \overline{\eta}}{l_c^2}, \quad f \equiv \frac{\langle \mu^2 \rangle}{\overline{\eta}^2} \tag{A 3}$$

and use the fact that $\langle \mu^2 \rangle = C/(2\tau_\eta)$ (recall that $\mu \equiv \eta - \overline{\eta}$), the dynamo numbers (equation 3.4) become

$$\mathcal{D}_1 \equiv \frac{9 f \tilde{\tau}}{4}, \quad \mathcal{D}_2 \equiv \frac{81 f \tilde{\tau}^2}{16} \tag{A 4}$$

Note that when $\tilde{\tau} \to 0$, $\mathcal{D}_1$ remains constant, while $\mathcal{D}_2 \to 0$. Here, f represents the strength of the fluctuations of η, while $\tilde{\tau}$ is a scaled measure of their correlation time.